Reinforcement corrosion: Limit states, reliability and modelling

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Reinforcement Corrosion: Limit States, Reliability and Modelling

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Abstract

An assessment of deteriorative effects on reinforced concrete structures is presented focusing mainly on the propagation of the reinforcement corrosion stage. It necessitates the utilization of relevant models and advanced simulation techniques to enable the probabilistic analysis of concrete structures and service life prognosis. Several analytical models for the initiation of concrete cracking, for crack width and for a decrease in the effective reinforcement cross-section are briefly presented. The described approach is applied to some illustrative examples showing the feasibility of predicting the development of reinforcement corrosion and its effect on the service life and/or reliability of the structure. The authors believe that such methodology supports the optimisation of design strategies as well as effective decision-making processes.

1. Introduction

Reinforced concrete is a widely used material in structures of such different types as bridges, buildings, underground structures and others. Damage due to reinforcement corrosion is recognized as one of the major causes of the deterioration of such structures with service life reduction and considerable additional costs as consequences. Cost-effective decisions need to be made when forecasting the time when an existing structure should be repaired or predicting the service life of a structure during the design stage. Both these activities require some knowledge about the corrosion mechanism and its progress over time, and about the relevant reliability level. The objective is to ensure the functionality and reliability of the structure over its entire service life in order to obtain an optimal result from the whole-life financial point of view. Note that the performance requirements of a structure are generally established by means of performance criteria and associated constraints related to service life and reliability; considering in this respect the reinforcement corrosion of a concrete structure it is evident that a general approach capable of modelling relevant deteriorative effects is needed. An extensive amount of research on these topics has been organized and reported over the last decades and has been published elsewhere – a detailed analysis is not feasible here; some of these studies are mentioned in the following text. Moreover, the majority of such works deal with the deterministic approach, although advanced and effective service life assessments have to be based on probabilistic analyses – this is also a topic dealt with in the presented paper.

Owing to either the carbonation of reinforced concrete or the ingress of chlorides into such a material, the depassivation of the reinforcing steel within it occurs (a process known as the *initiation period*, or more exactly *corrosion initiation*). This may be followed (in the presence of moisture and oxygen) by a steel corrosion process (known as the *corrosion propagation period*) – see (Tutti 1982). Suitable methods for the monitoring and reliable mathematical modelling of associated effects are needed – better knowledge in this respect provides the basis for a practical and proactive strategy when designing and maintaining concrete structures. This is also reflected in recent international standards and documents – e.g. (ISO 2008) and (*fib* 2010).

Frequently, the initiation of reinforcement corrosion is supposed to define the end of a structure's service life – see e.g. (JSCE 2007). However, in certain situations the corrosion propagation period may form a significant part of the total service life of a structure and the residual service life is taken as the time remaining before a crack of certain width develops at the surface of the concrete. This has been studied e.g. by (Ohtsu and Uddin 2008). Generally, the specified service life (which should be decided by the owner) defines the period in which the required performance is maintained (*fib* 2010).

This is why the present paper concentrates on the propagation period, i.e. reinforcement corrosion effects and relevant limit states suitable for design practice. In this respect, the chosen models of the degradation mechanisms are of the simplified type; their functionality is reinforced by the probabilistic approach and a user-friendly software tool.

Note that modelling of the initiation period and associated degradation mechanisms has been dealt with in many other works, e.g. (Papadakis *et al*. 1992; RILEM 1996; Engelund and Faber 2000; Teplý *et al*. 2010).

The goal of the paper is to present a tool for the full

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probabilistic analysis of concrete structures and for service life prognosis in the context of limit states associated with durability and reliability focused on the corrosion propagation stage. Note that the same approach and tools may be adopted for the initiation of the corrosion stage – see e.g. (Teplý *et al*. 2007 and 2012). Predictive models for material degradation are utilized together with advanced simulation techniques; thus the optimisation of design strategies as well as effective decision-making processes may be supported.

2. Limit states associated with the propagation period

Among the key factors influencing reinforcement corrosion in concrete are the presence of moisture and the ingress of oxygen from the air. The reader may find a detailed description of these factors in e.g. (Morigana 1988; Živica 1994; Escalante and Satoshi 1990). A number of models have been developed which can be roughly divided into two categories – empirical models and more complex electrochemical models. For the purpose of introducing a practical approach for the prognosis of the service life and relevant reliability level of concrete structures the simplest models are utilized in the present work. Inherent uncertainties in the material, technological and environmental characteristics of reinforced concrete structures have to be considered while solving such problems. To achieve this, a probabilistic approach has been utilized.

The limit state (LS) concept – in either its Serviceability limit state (SLS) or Ultimate limit state (ULS) variant – is applicable here, governed by a probability condition which in its general form reads:

$$P_f = P\left[A(t) \geq B(t)\right] < P_d \tag{1}$$

where P_d is the design (required) probability, t is time, A is the effect of the analysed action and B is the barrier. Generally, both A and B are time dependent and hence the probability of failure P_f is time dependent too. The combined effect of both structural performance and ageing should be considered wherever relevant. Time t_S corresponds to the limit given by eq. (1), i.e. it is the predicted service life; time t_D is the design service life. Both t_S and the deteriorative effect A are assessed by utilization of the appropriate degradation models, which apply a probabilistic approach. Note that the index of reliability β is frequently utilized in practice instead of the probability of failure P_f – see e.g. (ISO 1998; CEN 1990 2002).

The level of reliability in the context of durability should be left to the client's decision together with the definition of a target service life, creating in this way a necessary background for the making of critical decisions (e.g. financial optimisation) – see e.g. (*fib* 2010; Teplý 2007).

Concerning reinforcement corrosion and the appropriate LS, the following cases may be distinguished:

(i) The volume expansion of rust products develops tensile stresses in the surrounding concrete leading to concrete cracking (mainly affecting the concrete cover); for more details about the crack mechanism see e.g. (Ohtsu and Uddin 2008). In this case, B in Eq. (1) represents the critical tensile stress that initiates a crack in concrete at the interface with a reinforcing bar. A is the tensile stress in concrete at design service life t_D ;

(ii) alternatively, B is the critical crack width at the concrete surface and A is the crack width at the concrete surface generated by reinforcement corrosion at time t_D. When the progress of corrosion and consequently the opening of cracks continue, a network of cracks is propagated that possibly reaches the surface of the concrete cover. Together with cracks due to mechanical loading (Miyazato and Otsuki 2010), a crack network may form and lead to the separation of concrete elements. Such delamination is a complex effect depending e.g. on the diameters of reinforcing bars, their location, concrete quality, coarse aggregate size, cover, the type and amount of loading, and the configuration of the structure. Such a state is either an SLS or a ULS – depending on the location and the severity of this effect;

(iii) given a decrease in the effective reinforcement cross-section due to corrosion and excessive deformation, loss of bearing capacity and finally the collapse of the member/structure in question may occur. Such effects fall either into the SLS (deformation capacity) or ULS (load bearing capacity) categories; in this situation A is the actual (modelled) reinforcement cross-sectional area at time t_D and B is the minimum acceptable reinforcement cross-sectional area with regard to either an SLS or a ULS;

(iv) finally, some changes will occur in the characteristics of the bond between steel and concrete due to corrosion. This may lead either to an excessive deflection of the structure or to a loss of structural strength. The former appears to be a more critical effect (Zhang *et al*. 2009). The bond may be modelled by using the time function of bond stress vs. slip affected by the degree of reinforcement corrosion.

3. Software tool and models

3.1 Model selection

With respect to durability assessment it is important to develop a relevant approach, evaluation procedures and methods for service life prognosis. In this area, mathematical modelling is often a useful tool together with relevant limit states for the accomplishment of such a task. Several models may be available for the degradation process in question and the engineer can select a suitable one for each specific use. The main criteria in selecting the relevant degradation model for each specific use may be listed as follows:

1. the type of relevant limit state and exposure conditions;
2. the accuracy and relevance of the model when using the available data in relation to the required design exactness;
3. the type of representation (1D, 2D or 3D; spatial and/or temporal variability);
4. the level of physical sophistication (macro-, meso- or micro-level);
5. the level of mathematical involvement;
6. the possibility or feasibility of model combination or conditionality;
7. the availability of model data and their statistical characteristics, and/or the availability of relevant testing methods;
8. labor and/or time consumption;
9. level of model validation and calibration;
10. the type of concrete (e.g. High-Performance Concrete, Fibre-Reinforced Concrete, ...) and/or the type of structure;
11. the availability of efficient software tools.

For pragmatic reasons the model choice is frequently based on points 7 and/or 11.

There are several predictive computational models for the modelling of degradation caused by the corrosion of reinforcement. They are mainly heuristic and operate on a deterministic basis, using more or less simplified approaches and relatively easily obtainable data. As an example of a more complex model let us mention the model developed by (Isgor and Razaqpur 2006). It is a comprehensive finite element model for predicting the rate of steel corrosion in concrete structures considering processes which commence in the initiation stage, such as temperature, moisture, chloride ion and oxygen transport within concrete. It continues in the propagation stage and allows the model to include the effects of changes in exposure conditions; the corrosion rates on the surface of steel are calculated by solving Laplace's equation for electrochemical potential with the appropriate boundary conditions, including the relationship between over potential and current density for the anodic and cathodic regions. A non-linear solution algorithm is used. Evidently, such a model is not feasible for service life assessment in design practice and is not included in the authors' present work.

The simple models dealt with in the present contribution are summarized in **Table 1**. A common feature of all models is that the input data suffer from a particular type of uncertainty (known as *aleatory uncertainty* – associated with randomness, i.e. variability that represents natural randomness) and sometimes are not easily available. Because of this, the development of the applications utilizing stochastic methods is inevitable.

The authors of this paper and their co-workers have developed a software application called *FReET-D* in which all relatively well-known models are converted to the probabilistic format (input variables are treated as random variables) and summarized within the framework of a unified software environment. In *FReET-D* a combination of analytical models and simulation techniques has been amalgamated to form specialized software for assessing the potential degradation of newly designed as well as existing concrete structures (Teplý *et al*. 2007; Veselý *et al*. 2010). The included models for carbonation, chloride ingress and corrosion of reinforcement (among other phenomena) can be directly employed in the durability assessment of concrete structures in the form of a durability LS, i.e. the assessment of service life and of the level of the relevant reliability measure. Several features are offered including parametric studies and Bayesian updating. Altogether, 32 models are implemented as pre-defined dynamic-link library functions. In this paper only the included models for corrosion of reinforcement are presented.

FReET-D is actually a specialized module of *FReET* software (Novák *et al*. 2010). *FReET* probabilistic software allows simulations of uncertainties affecting an analyzed problem basically at the level of random variables (in civil/mechanical engineering these are typically material properties, loading, geometrical imperfections). Attention is given to those techniques that have been developed for the analysis of computationally in-

Table 1 Models for corrosion propagation in FReET-D.

Model notation	Source	Output	Comment
Corr1	Andrade *et al*. (1996) and Rodriguez *et al*. (1996)	Net rebar diameter at time *t*	Uniform type of corrosion
Corr2	Gonzalez *et al*. (1995)	Pit depth at time *t*	Pitting type of corrosion
Corr3	Val and Melchers (1998)	Net cross sectional area of rebar at time *t*	Pitting type of corrosion
Corr4	Liu and Weyers (1998)	Time to cracking due to corrosion	Crack initiation on the steel-concrete interface; uniform corrosion
Corr5	Li *et al*. (2006)	Crack width due to corrosion at time *t*	Crack width on concrete surface; uniform corrosion
Corr6	El Maaddawy and Soudki (2002)	Time to cracking due to corrosion	Crack initiation on the steel-concrete interface; uniform corrosion
Corr7	Du et al. 2005a, b; Cairns 2005; Yi *et al*. 2010	strength and ductility of corroded steel	Yield stress and ultimate stress - apparent values; strain at ultimate strength

tensive problems; nonlinear FEM analysis being a typical example. The stratified simulation technique Latin hypercube sampling (LHS) is used in order to keep the number of required simulations at an acceptable level. This technique can be used for both the random variable and random field levels. Statistical correlation is efficiently imposed by the stochastic optimisation technique known as simulated annealing, which is described by Vořechovský and Novák (2009). Sensitivity analysis is based on nonparametric rank-order correlation coefficients.

In the following sections the models implemented in *FReET-D* for corrosion of reinforcement in concrete are briefly described. For the purpose of this paper and for simple cross referencing the models are labeled as CorrX in the following sections, where X is an ordinal number. These models are summarized in **Table 1** and briefly described in the following text.

3.2 Corr1 - rebar diameter

This model by Andrade *et al*. (1996) and Rodriguez *et al*. (1996) is frequently used for the prediction of uniform corrosion. The formula for the time-related net rebar diameter $d(t)$ at exposure time t reads (see **Fig. 1a**):

$$d(t) = \begin{cases} d_i & t \le t_i \\ \psi\left[d_i - 0.0116\, i_{corr} R_{corr} (t - t_i)\right] & t_i < t \le t_i + \dfrac{d_i}{0.0116 i_{corr} R_{corr}} \\ 0 & t > t_i + \dfrac{d_i}{0.0116 i_{corr} R_{corr}} \end{cases} \tag{2}$$

where d_i is the initial bar diameter; $t = t_i + t_p$ where t_i is the time to corrosion initiation (steel depassivation), and t_p is the time of corrosion propagation. Parameter R_{corr} expresses the type of corrosion; ψ is the uncertainty factor of the model. For uniform corrosion the coefficient R_{corr} equals 2. The effects of chloride concentration, pH level or other conditions may also be described by the coefficient R_{corr} whenever applicable. The parameter i_{corr} is the current density; the mean value can be obtained by measurement or testing. Alternatively, there exist various analytical equations derived on the basis of experimental data. For example Li and Lawanwisut (2003) derived the formula that describes the dependence of i_{corr} on time, which is derived on the basis of experiments for RC flexural members. In (Vořechovská *et al*. 2009) the parameter i_{corr} was derived from experimental measurements as a function of chloride concentration for two different pH values of 9 and 12.5. It was recalculated from the corrosion rate found on the basis of experiments and was deliberately rearranged to transform it into the uniform type of corrosion. The pH values of 9 and 12.5 were chosen due to the fact that the pH value of the pore solution is about 12.5 in the case of non-carbonated concrete and pH = 9 in the case of carbonated concrete.

3.3 Corr2 - depth of pit

In cases of localised corrosion the following model can be taken into consideration. The studies by Gonzalez *et al*. (1995) show that the maximum rate of corrosion penetration in the case of pitting corrosion is about 4-8 times that of general corrosion. The depth of the pit $p(t)$ at time t can be estimated by the following equation (see **Fig. 1b**):

$$p(t) = \psi\left[0.0116\, i_{corr}\ R_{corr} (t - t_i)\right] \tag{3}$$

The meaning of the parameters is the same as in Eq. (2). The value of coefficient R_{corr} varies from 4 to 8 (Gonzalez *et al*. 1995); note that the effects of chloride concentration, pH level or other conditions may also be described by the coefficient R_{corr} whenever applicable. It is evident that the model given by Eq. (3) can serve as an estimate only. The value of $p(t)$ may serve as an input parameter for model Corr3.

3.4 Corr3 - net cross sectional area

An alternative means of modelling localised corrosion has been presented by Val and Melchers (1998). The section at the pits is predicted by a simplification in the hemispherical form. The net cross sectional area of a corroded rebar A_r at time t is calculated as the geometrical area of the grey colored part in **Fig. 1c**. The value of $p(t)$ may be obtained from model Corr2.

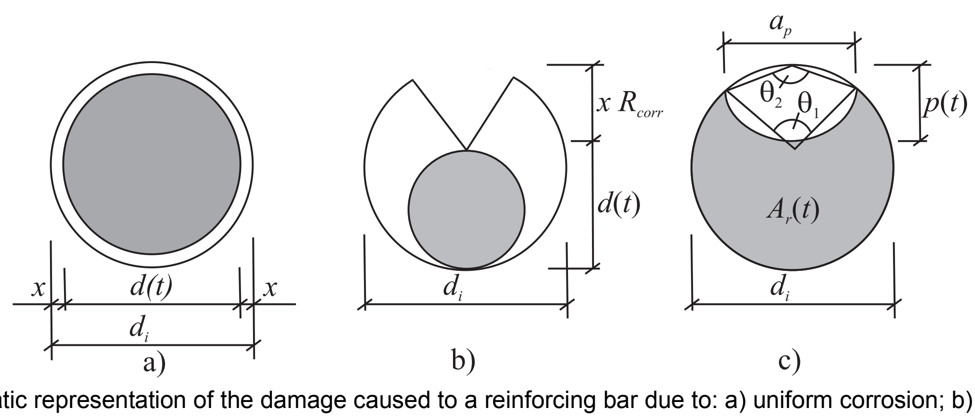

Fig. 1 Schematic representation of the damage caused to a reinforcing bar due to: a) uniform corrosion; b) and c) pitting corrosion.

It should be noted that pitting corrosion is highly localised on individual reinforcement bars. It is unlikely that many bars could be affected in the same section of a construction member; hence, pitting corrosion will not significantly influence the structural capacity in the early stage of corrosion.

3.5 Corr4 - time to cracking of concrete cover

The corrosion cracking model was proposed by Liu and Weyers (1998) to estimate the critical time to cracking $t_c = t_i + t_{p,cr}$ of concrete cover due to stresses resulting from the expansion of corrosion products from reinforcement. Within this paper only the modelling of $t_{p,cr}$ is utilized. It is assumed that uniform corrosion causes rust products to form uniformly around the steel surface. Three stages are considered in this model: (1) Free expansion – the porous zone around the reinforcement is filled by rust products. The total amount of corrosion products is less than the amount of corrosion products required to fill the porous zone around the steel-concrete interface W_P and so no stresses are exerted on the surrounding concrete; (2) Stress initiation – the total amount of corrosion products exceeds the amount of corrosion products needed to fill the porous zone W_P and expansive pressure is applied to the surrounding concrete; (3) Cracking – the total amount of corrosion products reaches the critical amount of corrosion products W_{crit} and cracking of the surrounding concrete is induced.

For a constant corrosion rate, the resulting time to cracking at the rust-concrete interface since corrosion initiation $t_{p,cr}$ can be derived as:

$$t_{p,cr} = \psi \frac{W_{crit}^2}{2k_p} \tag{4}$$

where k_p is related to the rate of metal loss and is expressed as:

$$k_p = 0.092\left(\frac{1}{\alpha}\right)\pi d_i i_{corr} \tag{5}$$

where α is the ratio of steel to the molecular weight of the corrosion products.

3.6 Corr5 - corrosion induced crack width

This model, proposed by Li *et al*. (2006), provides an estimation of corrosion-induced crack width – essential information for the prediction of the serviceability of corrosion-affected RC structures.

A concrete element with an embedded reinforcing steel bar can be modelled as a thick-walled cylinder – see (Bažant 1979; Pantazopoulou and Papoulia 2001; Tepfers 1979). This is shown schematically in **Fig. 2a**. Usually the porous zone thickness d_0 is constant once the concrete has hardened. When the reinforcing steel corrodes, its products fill the pore band completely. As the corrosion propagates in the concrete, a ring of corrosion products forms, the thickness $d_s(t)$ (**Fig. 2b**). The growth of the ring of corrosion products (known as a rust band) exerts outward pressure on the concrete at the interface between the rust band and the concrete. Under this expansive pressure the concrete cylinder undergoes three phases in terms of cracking: (i) no cracking; (ii) partially cracked; and (iii) completely cracked.

When the crack propagates to the concrete surface (case iii) the corrosion-induced concrete crack width can be expressed as follows (see also **Fig. 2d**):

$$w_c = \frac{4\pi d_s(t)}{(1-\nu_c)(x/y)^{\sqrt{\alpha_{stiff}}} + (1+\nu_c)(y/x)^{\sqrt{\alpha_{stiff}}}} - \frac{2\pi y f_t}{E_{ef}} \tag{6}$$

where $x = (d_i + 2d_0)/2$ and $y = a + (d_i + 2d_0)/2$, a is concrete cover, α_{stiff} is stiffness reduction factor, E_{ef} is the effective modulus of elasticity, f_t is the tensile strength of concrete and ν_c is the Poisson ratio of concrete.

3.7 Corr6 - time to cracking

A mathematical model for the prediction of time to cracking due to corrosion $t_c = t_i + t_{p,cr}$ was proposed by El Maaddawy and Soudki (2002) aiming for the further improvement of model Corr4. The authors developed a relationship between steel mass loss and the internal radial pressure caused by rust growth. The time to corrosion cracking is estimated on the basis of Faraday´s law.

Four basic assumptions were used in the currently-described model to determine the internal radial pressure caused by the expansion of corrosion products:

1) The corrosion products are formed uniformly around the steel reinforcing bar, which results in uniform expansive stresses around the steel bar.
2) There is a porous zone around the steel reinforcing bar which the corrosion products must first fill before they start to induce pressure in the surrounding

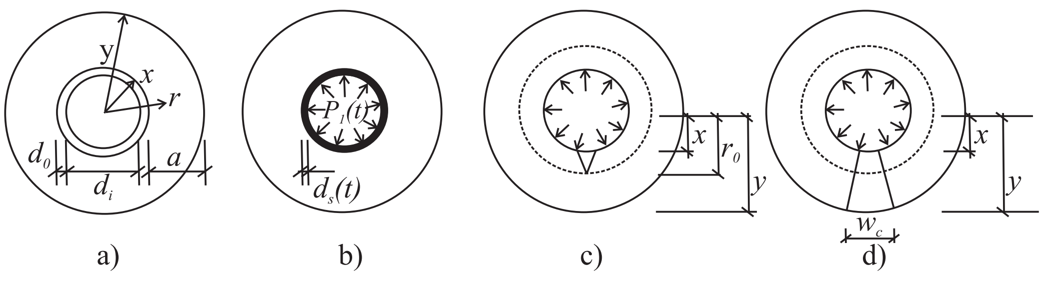

Fig. 2 Schematic diagram of the corrosion-induced concrete cracking process.

concrete.

3) The volume expansion caused by corrosion only creates strain in the concrete (i.e. strain in the steel is neglected).
4) The concrete around the steel reinforcing bar is modelled as a thick-walled cylinder with a wall thickness equal to the thinnest concrete cover. The concrete ring is assumed to crack when the tensile stresses in the circumferential direction at every part of the ring have reached the tensile strength of the concrete.

The time from corrosion initiation to corrosion cracking, $t_{p,cr}$, is obtained:

$$t_{p,cr} = \psi \frac{1}{365.25}\left[\frac{7117.5(d_i+2d_0)(1+\nu_c+k)}{i_{corr}E_{ef}}\right] \bullet \left[\frac{2af_t}{d_i}+\frac{2d_0E_{ef}}{(1+\nu_c+k)(d_i+2d_0)}\right] \quad (7)$$

where k is given by the following equation:

$$k = \frac{(d_i+2d_0)^2}{2a(a+d_i+2d_0)} \quad (8)$$

For the sake of completeness note that models Corr4, 5 and 6 are based on the thick-walled cylinder representation and so do not produce results realistic enough for cases when e.g. bars are positioned near the corner of a cross section or in cases of low cover. In such situations the corrosion effect may be better modelled using the Finite Element Method (FEM). Several papers have been presented utilizing such an approach for concrete cracking caused by steel corrosion, e.g. (Molina *et al.* 1993; Thoft-Christensen 2003; Pernica and Teplý 2005; Isgor and Razaqpur 2006; Vořechovský and Vořechovská 2010).

3.8 Corr7 - residual strength and ductility of corroded steel

Due to reinforcement corrosion the load-carrying capacity of a reinforced concrete member is reduced; if such corrosion appears, the decreased strength and ductility characteristics of the steel need to be assessed. Note the yield stress and ultimate stress are represented by the apparent values, i.e. based on the initial, non-corroded cross section of the bar. A practical model for the residual strength and ductility (Du *et al.* 2005a, b; Cairns 2005; Yi *et al.* 2010) of corroded steel (pitting or uniform type of corrosion) may be assessed by empirical correlation with cross section loss:

$$\begin{aligned} f_y &= (1.0-\alpha_y Q_{corr})f_{y0}\psi \\ f_u &= (1.0-\alpha_u Q_{corr})f_{u0}\psi \\ \varepsilon_u &= (1.0-\alpha_1 Q_{corr})\varepsilon_0\psi \end{aligned} \quad (9)$$

where f_y, f_u and ε_u are *y*ield strength, ultimate tensile strength and strain at ultimate strength after time t, while f_{y0}, f_{u0} and ε_0 represent *y*ield strength, ultimate tensile strength and strain at the ultimate strength of a non-corroded bar. Q_{corr} is the average cross section loss expressed as a percentage of the original cross section; α_y, α_u and α_1 are empirical coefficients. Note that the relevant Q_{corr} value (and its statistics) can be gained using the Corr1 or Corr2 model.

According to Du *et al.* (2005a, b) and Cairns (2005), an α_y or α_u value of 0.01 represents a uniform corrosion attack; values in excess of 0.01 represent the effect of pitting corrosion. For α_1 values from 0.017 to 0.06 were reported.

The influence of corroded steel on fatigue behavior is treated in a similar way in (Yi *et al.* 2010) as the corrosion damage of steel is more critical for RC structures under fatigue loading.

3.9 Influence of cracks

The influence of cracks due to shrinkage and mechanical load upon depassivation and/or corrosion of reinforcement should also be mentioned here since no general and well-verified model yet exists. According to fib Model Code (2010) the minimum structural reliability of a cracked concrete structure shall be of comparable magnitude to the minimum reliability of a comparable exposed uncracked structure, providing the limiting crack width is in the range of 0.2 to 0.4 mm.

An example of a model dealing with such a problem may nonetheless be cited, i.e. (Zhang *et al.* 2011), where a combined model that covers several factors (including load-induced cracks) is discussed with a focus on the chloride diffusion process for structures in marine environments.

4. Examples

4.1 Comparison of steel corrosion models

The above-mentioned models operate using different sets of input parameters. To compare them completely is not a simple or straightforward task. For this reason only three models are compared here – Corr4, Corr5 and Corr6, all in deterministic form only for the purpose of a clearer comparison. Corr4 and Corr6 enable the calculation of the time to crack initiation in concrete while Corr5 allows the tracking of the whole cracking process. As assumed in these models, concrete with an embedded reinforcing steel bar is modelled as a thick-walled cylinder. The models are compared in **Fig. 3a** via the dependence of r_c on d_s, where r_c is the radial distance from the steel-concrete interface to the outer face of the cracks and d_s is the thickness of the ring of corrosion products (see **Fig. 3b**). A time axis is also given in the figure. A full description of all input values can be found in **Table 2**.

In **Fig. 3a**, zero time is assumed to be the time of corrosion initiation. According to the Corr4, Corr5 and Corr6 models, cracks in concrete initiate in 2.10 (triangular point), 0.61 (rectangular point) and 0.27 (circular point) years, which corresponds to d_s = 0.0156, 0.0084

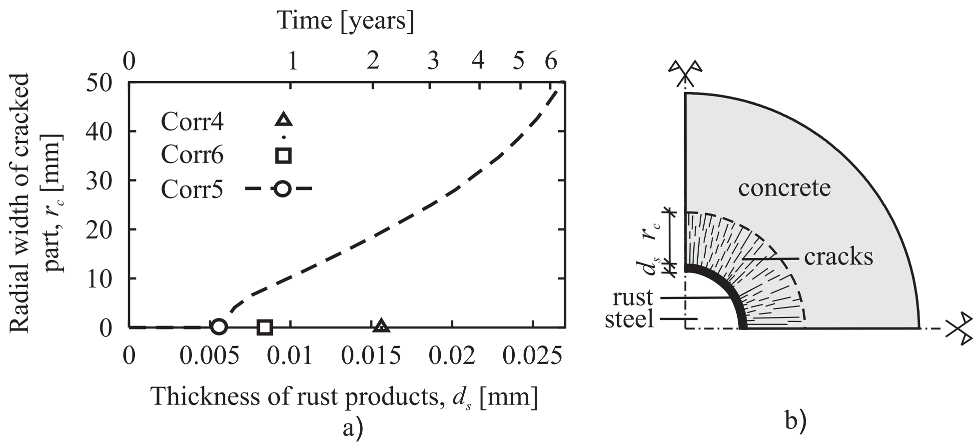

Fig. 3 (a) Comparison of models Corr4, Corr5 and Corr6; (b) a quarter of the section of the model.

Table 2 Input parameters for corrosion model comparison.

Input parameter	Nomenclature	Unit	Value	Model: Corr
Initial bar diameter	d_i	mm	25	4,5,6
Porous zone thickness	d_0	mm	0	4,5,6
Concrete cover	a	mm	50	4,5,6
Current density	i_{corr}	μA/cm^2	1	4,5,6
Specific gravity of rust	ρ_{rust}	kg/m^3	3925	4,5,6
Specific gravity of steel	ρ_{st}	kg/m^3	7850	4,5,6
Ratio of steel to rust molecular weight	α	-	0.622	4,5,6
Tensile strength of concrete	f_t	MPa	2.4	4,5,6
Elastic modulus of elasticity of concrete	E_c	GPa	30.5	4,5,6
Poisson´s ratio of concrete	ν_c	-	0.24	4,5,6
Creep coefficient	φ_{cr}	-	2	4,5,6
Material constant	γ	-	1	5

and 0.0056 mm, respectively. These d_s values, which correspond to times of crack initiation, agree with linear tangential stresses in concrete (at the steel-concrete interface) of 9.60, 5.17 and 3.41 MPa, respectively. Note that the input tensile stress of concrete has been taken to be equal to 2.4 MPa.

4.2 Critical crack width on the concrete surface

A parametric study utilizing the Corr5 model is demonstrated in this example. The limit state condition is constructed in this case in the form:

$$P_f(t_D) = P\left\{w_{cr} - w_a\left(t_D\right) \le 0\right\} \le P_d \qquad (10)$$

where w_{cr} is the limit value of a crack width equal to 0.3 mm – one of the essential limits for the serviceability assessment of corrosion-affected RC structures recommended in CEN (2003). The actual corrosion-induced crack width w_a over time is computed according to Eq. (6). In this example, time t_D represents the propagation period only; to gain a service life prediction the appropriate initiation period must be added.

All input information is listed in **Table 3**. Note that the adopted concrete is approximately of class C35/40, which is relevant e.g. to exposition class XD3 according to CEN (2003).

In **Fig. 4** the results of statistical analysis are depicted, i.e. the values of crack width on the concrete surface after 15 years of steel corrosion development as the function of concrete cover a. Next, **Fig. 5** shows the reliability index β vs. concrete cover for three different propagation time values; the limit value of β = 1.5 prescribed typically for SLS is also shown in the figure. It appears that e.g. for t_D = 25 years the cover should be greater than 50 mm to satisfy the serviceability requirements; for t_D = 15 years 30 mm of cover would be satisfactory.

5. Concluding remarks

- The use of modelling in the field of reinforcement corrosion in RC structures has recently increased in the context of the solving of everyday design problems. Nevertheless, it is recommended that users be aware of the conditions under which the model in question was developed – see comments by Otieno *et al*. (2011) or Ahmad (2003). The final remarks of (fib Bulletin 59 2011) may be cited in this respect: "…

Table 3 Input parameters for crack width analysis.

Input parameter	Nomenclature	Unit	Distribution	Mean	COV
Initial bar diameter	d_i	mm	Normal	20	0.02
Porous zone thickness	d_0	mm	Deterministic	0.0125	
Concrete cover	a	mm	Deterministic	30-70	
Time of exposure	t	years	Deterministic	15, 20, 25	
Current density	i_{corr}	μA/cm^2	Normal	1.5	0.2
Specific gravity of rust	ρ_{rust}	kg/m^3	Normal	3600	0.02
Specific gravity of steel	ρ_{st}	kg/m^3	Normal	7850	0.01
Ratio of steel to rust molecular weight	α	-	Deterministic	0.57	
Tensile strength of concrete	f_t	MPa	Lognormal (2 par)	3.3	0.12
Elastic modulus of elasticity of concrete	E_c	GPa	Lognormal (2 par)	27	0.08
Poisson ratio of concrete	ν_c	-	Deterministic	0.18	
Creep coefficient	φ_{cr}	-	Deterministic	2	
Material constant	γ	-	Deterministic	1	
Uncertainty factor of model	ψ	-	Lognormal (2 par)	1	0.15

models, methods and procedures are of value when used properly and adjusted to the structural situation. The sensitive selection of input data … adequate limit states for decision making are therefore necessary".

- The scheme of the presented methodology for the assessment of a degrading concrete structure is as follows: information/data about the structure → exposure conditions → choice of limit state → choice of model → obtaining of relevant input data values (including their statistical parameters) → statistical and reliability analysis (possibly also sensitivity analysis) → service life and reliability index assessment → evaluation of results and appropriate decision making.
- A broader choice of models is useful e.g. due to problems with the availability of statistical data for the input variables of some models.
- The usefulness of effective degradation modelling and hence reliable design/assessment for durability may enhance advanced design practice, decision processes and, as a consequence, bring positive financial and sustainability impacts.

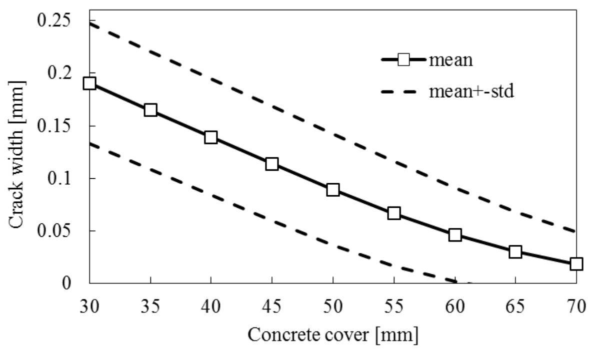

Fig. 4 Crack widths on the concrete surface after 15 years of steel corrosion development.

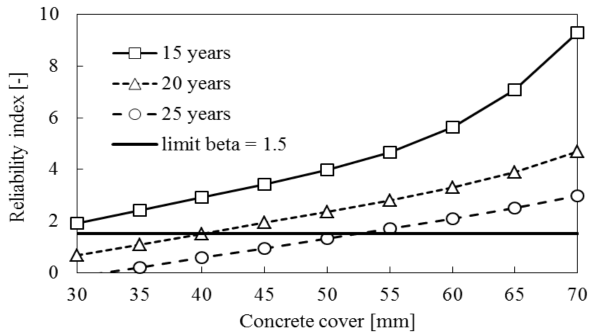

Fig. 5 Reliability index β after 15, 20 and 25 years of steel corrosion development (the condition given by Eq. 10).

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References

Ahmad, S., (2003). "Reinforcement corrosion in concrete structures, its monitoring and service life prediction: a review." *Cement & Concrete Composites*, 25, 459-471.

Andrade C., Sarria J. and Alonso C., (1996). "Corrosion rate field monitoring of post-tensioned tendons in contact with chlorides." In: *Durability of Building Materials and Components 7*, Vol. 2, Stockholm, 959-967.

Bažant, Z. P., (1979). "Physical model for steel corrosion in concrete sea structures - Theory." *Journal of Structural Division*, ASCE, 105(ST6), 1137-1153.

Cairns, J., Plizzari, G. A., Du, Y., Law, D. W. and Franzoni, Ch., (2005). "Mechanical properties of corrosion damaged reinforcement." *ACI Materials*

Journal, 102(4), 256-265.

CEN, (2002). "*Basis of structural design*". EN 1990, Eurocode 0.

CEN, (2003). "*General rules and rules for buildings.*" EN 1992-1-1 Design of concrete structures, Part 1-1, Eurocode 2.

Du, Y. G., Clark, L. A. and Chan, A. H. C., (2005a). "Residual capacity of corroded reinforcing bars." *Magazine of Concrete Research*, 57(3), 135-147.

Du, Y. G., Clark, L. A. and Chan, A. H. C., (2005b). "Effect of corrosion on ductility of reinforcing bars." *Magazine of Concrete Research*, 57(7), 407-490.

El Maaddawy, T. and Soudki, K., (2007). "A model for prediction of time from corrosion initiation to corrosion cracking." *Cement and Concrete Composites*, 29(3), 168-175.

Engelund, S. and Faber, M. H., (2000). "Development of a code for durability design of concrete structures." In: Melchers and Stewart, Eds. *Applications of Statistics and Probability*, Balkema, Rotterdam.

Escalante, E. and Satoshi, I., (1990). "Measuring the rate of corrosion of steel in concrete." *Corrosion Rates of Steel in Concrete*, ASTM STP 1065, 86-102.

fib, (2010). "*fib draft model code*". *fib* Bulletins No. 55 and 56. Lausanne, Switzerland.

fib Bulletin No. 59, (2011). "*Condition control and assessment of reinforced concrete structures.*" Lausanne, Switzerland.

Gonzales, J. A., Andrade, C., Alonso, C. and Feliu, S., (1995). "Comparison of rates of general corrosion and maximum pitting penetration on concrete embedded steel reinforcement." *Cement and Concrete Research*, 25(2), 257-264.

Isgor, O. B. and Razaqpur, A. G., (2006). "Modelling steel corrosion in concrete structures." *Materials and Structures*, 39, 291-302.

ISO 2394, (1998). "*General principles on reliability for structures.*" ISO.

ISO 13823, (2008). "*General Principles in the Design of Structures for Durability.*" ISO.

JSCE, (2007). "*Standard Specification for Concrete Structures - Design.*" Japan Society of Civ. Engrs.

Li, C. Q., Melchers, R. E. and Zheng, J. J., (2006). "An analytical model for corrosion induced crack width in reinforced concrete structures." *ACI Structural Journal*, 103(4), 479-482.

Li, C. Q. and Lawanwisut, W., (2003). "Serviceability assessment of reinforced concrete structures in marine environments." In: *Life prediction and aging management of Concrete structures*, 2nd RILEM Workshop, Paris, 127-136.

Liu, Y. and Weyers, R. E., (1998). "Modeling the time-to-corrosion cracking in chloride contaminated reinforced concrete structures." *ACI Material Journal*, 95(6), 675-681.

Miyazato, S. and Otsuki, N., (2010). "Steel Corrosion Induced by Chloride or Carbonation in Mortar with Bending Cracks or Joints." *Journal of Advanced Concrete Technology*, 8(2), 135-144.

Molina, F. J., Alonso, C. and Andrade, C., (1993). "Cover cracking as a function of rebar corrosion: Part 2 - Numerical model." *Materials and Structures*, 26, 532-548.

Morigana, S., (1988). "*Prediction of service lives of reinforced concrete buildings based on the corrosion rate of reinforcing steel.*" Special Report No. 23, Institute of Technology, Shimizu Corporation, Tokyo, Japan.

Novák, D., Vořechovský, M. and Rusina, R., (2010). "*User´s and Theory Guides.*" FReET v.1.5 - program documentation. Brno/Červenka Consulting, Czech Republic, http://www.freet.cz.

Ohtsu, M. and Uddin, F. A. K. M., (2008). "Mechanisms of corrosion-induced cracks in concrete at meso- and macro-scales." *Journal of Advanced Concrete technology*, 6(3), 419-428.

Otieno, M. B., Beuhausen, H. D. and Alexander, M. G., (2011). "Modelling corrosion propagation in reinforced concrete structures - a critical review." *Cement & Concrete Composites*, 33, 240-245.

Pantazopoulou, S. J. and Papoulia, K. D., (2001). "Modeling Cover-Cracking due to Reinforcement Corrosion in RC Structures." *Journal of Engineering Mechanics*, ASCE, 127(4), 342-351.

Papadakis, V. G., Fardis, M. N. and Vayenas, C. G., (1992). "Effect of composition, environmental factors and cement-lime mortar coating on concrete carbonation." *Materials and Structures*, 25(149), 293-304.

Pernica, F. and Teplý, B., (2005). "FE modelling of reinforcement corrosion in concrete." In: Vejvoda, S., Ed. *Koroze*, Brno, Czech Republic, 178-182.

RILEM Report 14, (1996). "Durability Design of Concrete Structures." In: Sarja, A. and Vesikari. E., Eds. E & FN SPON, London.

Rodriguez, J., Ortega, L.M., Casal, J. and Diez, J.M., (1996). "Corrosion of reinforcement and service life of concrete structures." In: *Durability of Building Materials and Components 7*, Vol. 1, Stockholm, 117-126.

Tepfers, R., (1979). "Cracking of concrete cover along anchored deformed reinforcing bars." *Magazine of Concrete Research*, 31(106), 3-12.

Teplý, B., (2007). "Durability, reliability, sustainability and client's decision". In: *CEBS 07 Prague Conference*, Prague, Vol. 1, 430-436.

Teplý, B., Chromá, M. and Rovnaník, P., (2010). "Durability assessment of concrete structures: reinforcement depassivation due to carbonation." *Structure and Infrastructure Engineering*, 6(3), 317-327.

Teplý, B., Chromá, M., Rovnaník, P. and Novák, D., (2012). "The role of modeling in the probabilistic durability assessment of concrete structures." In: *Life-Cycle and Sustainability of Civil Infrastructure Systems* - Strauss, Frangopol & Bergmeister (Eds).

Taylor & Francis Group, London, 876-882.

Teplý, B., Matesová, D., Chromá, M., Rovnaník, P., (2007). "Stochastic degradation models for durability limit state evaluation: SARA - part VI." In: *3rd International Conference on Structural Health Monitoring of Intelligent Infrastructure*, Vancouver, British Columbia, Canada, p. 187 (Abstract), full length paper on CD-ROM.

Thoft-Christensen, P., (2005). "Service life definitions based on corrosion crack width." In: G. Augusti, G.I. Schueller, M. Ciampoli, Eds., *ICOSSAR 2005*, Milpress, Rotterdam, 1391-1398.

Tutti, K., (1982). "*Corrosion of steel in concrete.*" Swedish Cement and Concrete Research Institute, Stockholm.

Val, D. and Melchers, R. E., (1998). "Reliability analysis of deteriorating reinforced concrete frame structures." *Structural safety and reliability*, Balkema, Rotterdam, Vol. 1, 105-112.

Vořechovská, D., Chromá, M., Podroužek, J., Rovnaníková, P. and Teplý, B., (2009). "Modelling of Chloride Concentration Effect on Reinforcement Corrosion." *Computer-Aided Civil and Infrastructure Engineering*, 24, 446-458.

Vořechovská, D. and Vořechovský, M., (2010). "Modeling of reinforcement corrosion in concrete." In: B. H. Oh *et al*., Eds. *7th International Conference on Fracture Mechanics of Concrete and Concrete Structures*, FraMCoS-7, Jeju, KOREA, p. 929-935.

Vořechovský, M. and Novák, D., (2009). "Correlation control in small sample Monte Carlo type simulations I: A Simulated Annealing approach." *Probabilistic Engineering Mechanics*, 24(3), 452-462.

Wei-Jian Yi, Sashi K. Kunnath, Xiao-Dong Sun, Cai-Jun Shi and Fu-Jian Tang, (2010). "Fatigue Behavior of Reinforced Concrete Beams with Corroded Steel Reinforcement." *ACI Structural Journal*, 107(5), 526-533.

Zhang, R., Castel, A. and Francois, R., (2009). "Serviceability Limit State criteria based on steel-concrete bond loss for corroded reinforced concrete in chloride environment." *Materials and Structures*, 42, 1407-1421.

Zhang, X., Zhao, Y., Xing, F. and Lu, Z., (2011). "Coupling effects of influence factors on probability of corrosion initiation time of reinforced concrete." *J. Cent. South Univ. Technol.*, 18, 223-229.

Živica, V., (1994). "Mathematical modeling of concrete reinforcement corrosion rate and the possibilities of service life prediction of reinforced concrete, Part I - Influence of environmental relative humidity and time." *Building Research Journal*, 42(3), 185-198.