A PRESTRESSED BRIDGE BEAM UNDER CORROSION ATTACK – PROBABILISTIC MODELLING

Dita Matesová, Jan Podroužek, Zbyněk Keršner, Břetislav Teplý

Summary

Presented here is an assessment of the limit state conditions of a prestressed bridge beam with its prestressing steel affected by corrosion. The proposed approach is based on probabilistic methods and simulation techniques, taking into account the uncertainties of the relevant variables while modelling hydrogen induced stress-corrosion cracking (SCC). The software package FReET-D [1, 2] for the stochastic assessment of degradation effects in reinforced concrete structures is utilized for SCC modelling and service life assessment. The deterministic model is based on the fracture mechanics approach. As an illustrative example, a precast prestressed bridge girder damaged by SCC is presented.

Keywords: Stress-corrosion cracking, analytical model, fracture mechanics, reliability, software

1 Introduction

In the last decade, considerable progress has been made in the realistic determination of the actual load carrying capacity and reliability of existing bridge structures. In this context, the deterioration of concrete due to corrosion of mild or prestressing reinforcement is a serious effect influencing the service life and reliability of concrete structures. Furthermore, a corrosion rate may arise in older bridge structures with quenched and tempered prestressing wires that are sensitive to hydrogen induced stress corrosion cracking (SCC). Despite the intensive research devoted to SCC, its underlying mechanisms are not yet fully understood as it depends upon complex chemical and mechanical interactions. In such cases large uncertainties are related to the deterioration of the materials, and consequently, structural member behaviour. Thus, the assessment of the load carrying capacity, serviceability, durability and reliability of a structure should be based on advanced methods taking into account the above mentioned uncertainties.

Computational models provide useful information about a structure and its behaviour in various phases of its life-time. This is important especially when the experimental investigation of the existing structure is unsuitable or even impossible. Prestressed bridges damaged by the hydrogen induced SCC of their wires may represent such a case. The effect of SCC cannot be simply described as a chemical or a mechanical effect. This paper presents the linear elastic fracture mechanics approach and the stochastic simulation technique applicable for the assessment of this problem.

2 FReeT-D software

Metals can be generally subjected to various forms of corrosion. In our case, the interaction is studied of corrosion and mechanical stress leading to failure due to cracking. The development of damage due to the interaction of corrosion and mechanical stress may be faster than other types of deterioration. The consequences of SCC may be costly and destructive. Unfortunately, research concerning SCC modelling is rather rare and engineering tools suitable for the handling of such situations do not yet exist (to the best knowledge of the authors). Let us note that a stochastic nonlinear assessment of a precast prestressed concrete bridge girder has been performed recently [3] using the ATENA [4] and FReeT [5] software packages that communicate through the SARA shell (*S*tructural *A*nalysis and *R*eliability *A*ssessment). This package was specifically designed for the purpose of reliability assessment of the nonlinear behaviour of reinforced concrete structures. Results of numerical simulations of the prestressed bridge girder with ad-hoc corroded reinforcement were presented in [6] showing decreases in the load carrying capacity and in the reliability index compared to a beam with prestressing steel without corrosion. However, no degradation model was utilized in that case.

The aim of this paper is to refer to an effective mathematical model describing SCC in prestressed steel wires. This model has been implemented into the multipurpose probabilistic software FReeT-D [1, 2] (*F*easible *Re*liability *E*ngineering *T*ool for *D*egradation effects assessment) which is based on FReeT [5]. FReeT-D involves 26 degradation models in the form of *dll* functions.

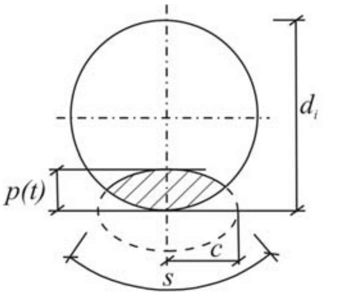
3 SCC modelling

Degradation models are generally time dependent mathematical functions describing the process of degradation during exposure to environmental and operational loading. While formulating the SCC model, the following basic assumptions were adopted: (i) although the real problem is practically a three-dimensional one, a two-dimensional approach is proposed for the sake of simplicity and computational efficiency; (ii) the corrosion develops in the form of pits that are considered to be stress concentrators; (iii) the position and range of pitting corrosion is random and the worst pit is taken into account here; and (iv) the geometry of the crack/pit is considered to be semi-elliptical and loaded via mode I opening.

The fundamentals of the model are adopted from [7]. The model proposed in this paper differs in the geometry function F_I evaluation, and has been randomized by the authors. As already mentioned, the implemented model is based on the fracture mechanics approach and thus is determined only with the geometry of reinforcement and the corrosion pit (see Fig. 1), the exposure time, and the actual uniform tension. The time dependency is included through the depth of the pit $p(t)$ [mm] (Fig. 1) that can be expressed as [8]:

$$p(t) = \psi\left(0.0116\ i_{corr}\ R_{corr}\left(t - t_i\right)\right) \tag{1}$$

where $t = t_i + t_p$ (see Fig. 2), t_i [years] is the time to corrosion initiation, i.e. steel depassivation, and may be gained using appropriate models for carbonation or chloride ingress, see e.g. [9], and t_p [years] is the time of corrosion propagation. ψ is the uncertainty factor of the model [–], i_{corr} is the corrosion current [μA/cm^2] and R_{corr} is a coefficient that expresses the corrosion type. The constant 0.0116 is a conversion factor from μA/cm^2 to mm/years under the assumptions that steel (Fe) has n = 2 (number of electrons freed by the corrosion reaction), M = 55.85 g (atomic mass) and d = 7.88 g/cm^3.



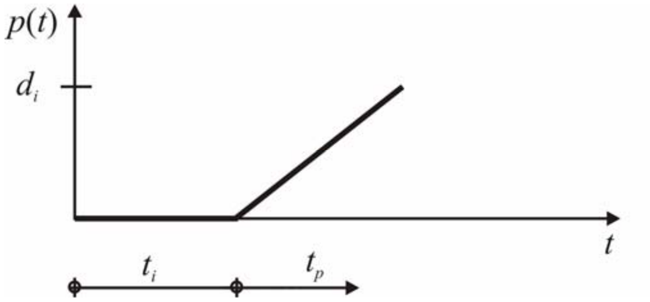


Fig. 1 The geometry of the prestressing wire and the corrosion pit (left)
Fig. 2 Exposure time period vs. corrosion development (right)

The output value of the proposed model, the stress intensity factor K_I [MPa.m$^{1/2}$] is defined as:

$$K_I = \psi F_I\left(t, d_i, c\right)\sigma\sqrt{s\left(d_i, s_p, t\right)} \tag{2}$$

A key parameter is the relevant geometry function F_I designed according to [10], plotted in Fig. 3 and defined as:

$$F_I = 1.76\exp\left(-1.054 s_p + 0.7072\left(\frac{2p(t)}{d_i}\right)^2\right) + 0.5548\ln(s_p) \tag{3}$$

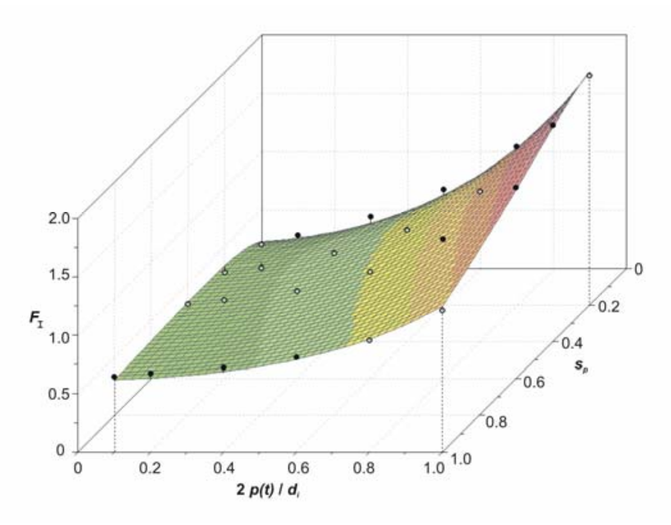


Fig. 3 Geometry function surface (Eq. 3)

In Eqs. (2) and (3) d_i is the initial wire diameter, s is the arc of a circle length given by the circle – ellipse intersection (see Fig. 1) and σ [MPa] represents the uniform tension in the prestressed wire considered as time independent for the sake of simplicity. The relationship between the ellipse's primary half-axe, c and the secondary half-axe that is the crack depth $p(t)$ (Fig. 1), is bonded by a shape parameter $s_p = p(t)/c$. The illustration of the influence of s_p on F_I is shown in Figs. 3 and 4.

4 Limit-state function

The model presented in section 3 may be effectively utilized while evaluating the limit state condition:

$$P_f(t_D) = P\left\{K_{ISCC} - K_I\left(t_D\right) \le 0\right\} \le P_d \tag{4}$$

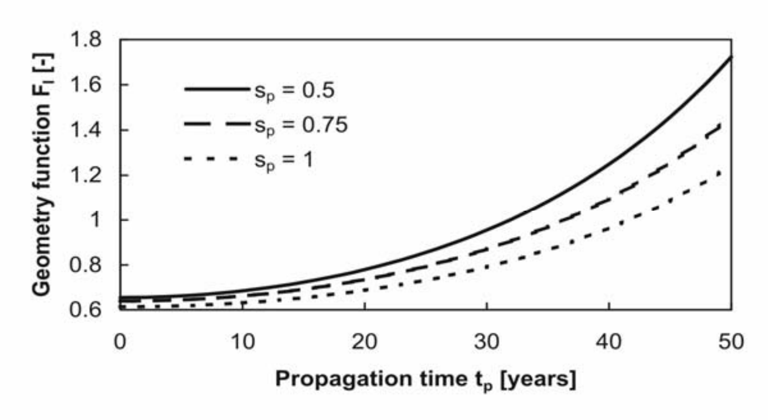


Fig 4 Geometry function F_I vs. propagation time t_p for d_i = 20mm, and the rate of the pit depth increase $p(t)$ = 0.2 mm/year

where t_D is the design (target) service life and K_{ISCC} is the threshold stress intensity factor for stress corrosion cracking. When K_I reaches the value of K_{ISCC}, the growth of a crack becomes unstable.

5 Illustrative examples

***Example 1*:** The analysis of the condition (4) has been performed for the following input data: d_i = 15.24 mm (=0.6 in), σ = N(1000; 80) MPa, s_p = 0.75 and ψ = LN(1; 0.15). The barrier K_{ISCC} was assumed to have Normal probability distribution with the mean values 40, 50 and 60 MPa·m$^{1/2}$ and COV = 0.1 [7]. Pitting corrosion rate was assumed to be 0.2 mm/year. This value was determined on the basis of Eq. (1) for the following inputs: i_{corr} = 3 µA/cm^2, R_{corr} = 6 and ψ = 1. The chosen values of i_{corr} and R_{corr} indicate a high rate and the pitting type of corrosion, respectively. The influence of the K_{ISCC} value on the probability of the failure of a single wire, Eq. (4), vs. propagation time t_p is plotted in Fig. 5.

***Example 2*:** As an application example, a precast prestressed bridge girder damaged by hydrogen induced SCC is simulated. The effect of corrosion on the load bearing capacity of the girder has been studied previously in [6]; the dimensions of the girder are similar to those of the AASHTO Type IV, capable of spanning distances in the 21 to 30 m range [11]. The influence of a rupture of 6 prestressing tendons out of a total of 26 due to SCC on the ultimate load and the probability of failure was shown in [6], utilizing nonlinear FE analysis and probabilistic assessment. From the study results, a 15% decrease in the bearing capacity can be compared to that of the girder with all prestressing tendons (the index of reliability β being equal to 3.8, as prescribed by EN 1990 as the target value at the 50-year reference period). No consequences for service life were studied in [6] as no time dependent model was available. To complete this is the reason why the same girder is investigated in the present study using the SCC model described in the previous sections.

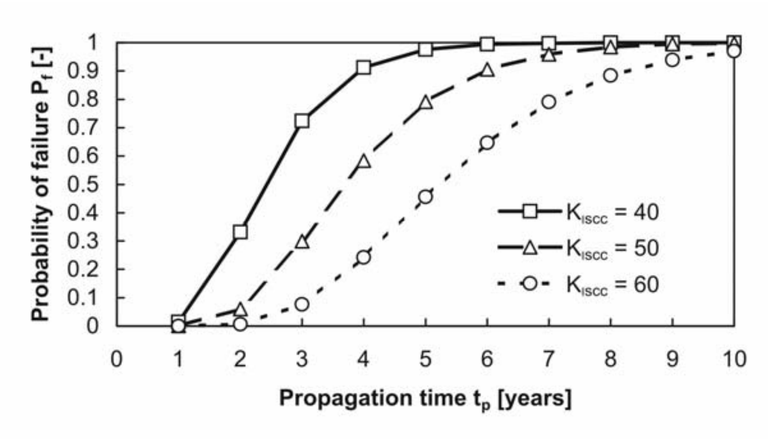


Fig. 5 The probability of failure of a single wire, Eq. (4), vs. propagation time t_p for various values of threshold K_{ISCC} [MPa·m$^{1/2}$]

Considering a rather extreme situation (in practice not likely to occur) – the rupture of 6 prestressing tendons at the same cross section at the same time – the propagation time t_p has been assessed. The values input to Eq. (1) are: $i_{corr} = 3$ µA/cm^2, $R_{corr} = 4$ and $\psi = 1$; to Eq. (2): d_i = N(11.2; 0.112) mm, σ = N(1020; 102) MPa, s_p = N(0.75; 0.075) and ψ = LN(1; 0.15).

First, the value of *p(t)* is studied, Eq. (1). The resulting corrosion rate used further for the SCC evaluation is 0.14 mm/year. In the next step the time to failure of the 6 prestressing tendons is calculated. Tensile stress σ is taken from the nonlinear numerical simulation [6] for the reliability level $\beta = 3.8$. The threshold value of the stress intensity factor K_{ISCC} = 60 MPa·m$^{1/2}$. The results are plotted in Fig. 6. It follows that the mean critical propagation time t_p is about 7 years.

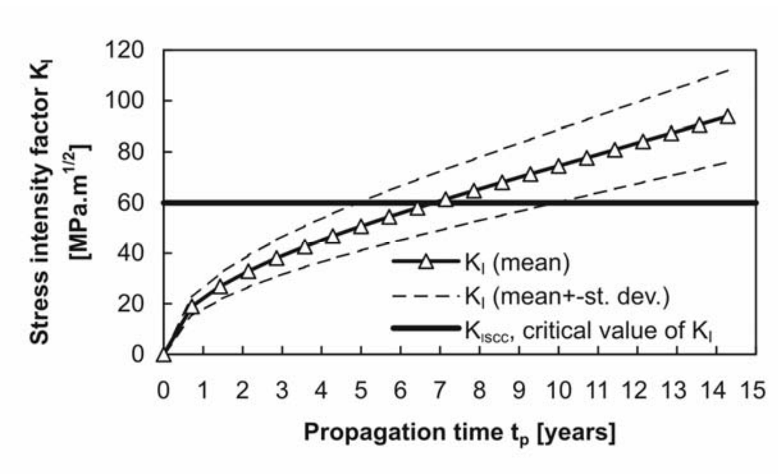


Fig. 6 Stress intensity factor K_I vs. propagation time t_p

The initiation time t_i has been studied under the assumption that chloride ingress is responsible for the steel's depassivation. The model proposed by Papadakis et al. [12] implemented in FReET-D [1, 2] was used. The description of this model is out of the scope of this text; the resulting time to corrosion initiation has been found to be t_i = 30 years.

In this way, the partial results gained in [6], i.e. the decrease in the load-bearing capacity to 85%, are completed by the assessment of the time $t_S = t_i + t_p$ = 37 years.

Acknowledgement
This outcome has been achieved with the financial support of the Czech Ministry of Education, project No. 1M0579, within the activities of the CIDEAS research centre.

References

[1] Teplý, B., Chromá, M., Matesová, D. & Rovnaník, P., (2006): "FReET-D. Program Documentation, Part 1, Theory".

[2] Veselý, V., Teplý, B., Chromá, M. & Matesová, D., (2006): "FReeT-D. Program Documentation, Part 2, User Manual".
[3] Pukl, R., Novák, D. & Bergmeister, K., (2003): "Reliability assessment of concrete structures". In Computational Modelling of Concrete Structures, edit. Bičanič et al., 2003 Swets & Zeitlinger, Lisse, p. 551-558.
[4] Červenka, V., Jendele & L., Červenka, J., (2005): "ATENA Program Documentation, Part 1 Theory", Červenka Consulting, Prague.
[5] Novák, D. & Vořechovský, M., (2006): "FReeT. Technical report", Institute of Engineering Mechanics, Faculty of Civil Engineering, Brno University of Technology, Czech Republic. Program documentation - Part 1 - Theory.
[6] Teplý, B., Lehký, D., Novák, D., Eichinger, E.M., Kolleger, J., Pukl, R., Červenka, V. & Stewart, M.G., (2003): "Prestressed bridges under corrosion attack". In Proc. of Euro-C 2003, p. 551-558.
[7] Izquierdo, D., Andrade, C. & Tanner, P., (2001): "Reliability Analysis of Corrosion in Post-tensional Tendons: Case Study", Proceedings of the international conference "Safety, Risk, Reliability – Trend in Engineering", Malta, p. 1001-1006.
[8] Gonzales, J.A., Andrade, C., Alonso, C. & Feliu, S., (1995):"Comparison of rates of general corrosion and maximum pitting penetration on concrete embedded steel reinforcement", Cement and Concrete Research, 25(2), p.257-264.
[9] Matesová, D. et al, (2006): "Tools for assessment of durability of concrete structures". Proc. of *PPK2006 – Probability of structural failures* (edit. S. Vejvoda), Brno, (in Czech), in print.
[10] Tada, H., Paris, P.C. & Irwin, G.R., (2000): "The Stress Analysis of Cracks Handbook". Third edition, ASME, New York.
[11] Taly, N., (1998): "Design of Modern Highway Bridges". McGraw-Hill, New York.
[12] Papadakis, V.G., Roumeliotis, A.P., Fardis, C.G. & Vagenas, C.G., (1996): "Mathematical modelling of chloride effect on concrete durability and protection measures". In Proc. of International Conference on Concrete in the Service of Mankind (Concrete Repair, Rehabilitation and Protection – edited by Dhir, R.K. and Jones M.R.), Dundee, Scotland, UK, p. 165-174.

Ing. Dita Matesová, PhD.
Jan Podroužek
Ing. Zbyněk Keršner, CSc.
Prof. Ing. Břetislav Teplý, CSc.
✉ Brno University of Technology
Faculty of Civil Engineering
Veveří 331/95
602 00 Brno, Czech Republic
☎ +420 5 4114 7368
☺matesova.d@fce.vutbr.cz
☺limajuliet@seznam.cz
☺kersner.z@fce.vutbr.cz
☺teply.b@fce.vutbr.cz
URL http://www.fce.vutbr.cz